

# Right Triangle Trigonometry

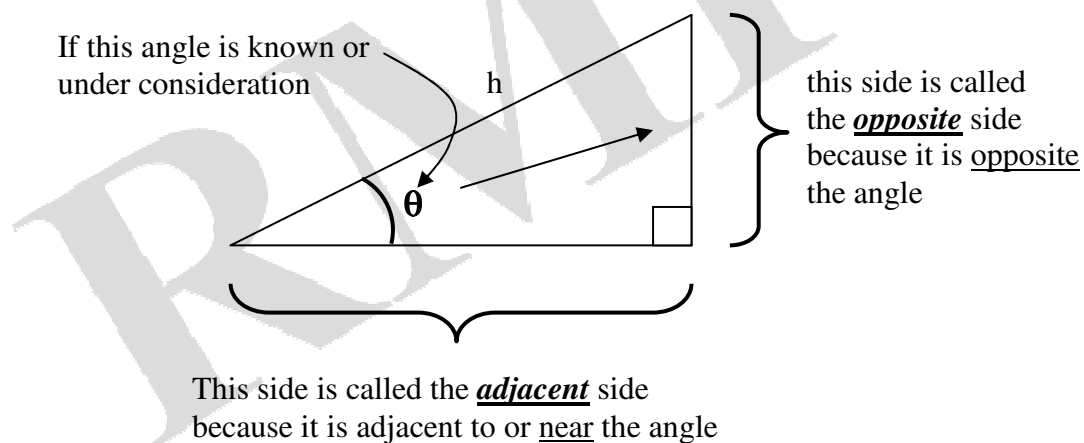
Trigonometry is a branch of mathematics involving the study of triangles, and has applications in fields such as engineering, surveying, navigation, optics, and electronics. Also the ability to use and manipulate trigonometric functions is necessary in other branches of mathematics, including calculus, vectors and complex numbers.

## Right-angled Triangles

In a right-angled triangle the three sides are given special names.

The side opposite the right angle is called the **hypotenuse** (h) – this is always the longest side of the triangle.

The other two sides are named in relation to another known angle (or an unknown angle under consideration).



## Trigonometric Ratios

In a right-angled triangle the following ratios are defined

$$\sin \theta = \frac{\text{opposite side length}}{\text{hypotenuse length}} = \frac{o}{h} \qquad \text{cosine } \theta = \frac{\text{adjacent side length}}{\text{hypotenuse length}} = \frac{a}{h}$$

$$\text{tangent } \theta = \frac{\text{opposite side length}}{\text{adjacent side length}} = \frac{o}{a} \qquad \text{where } \theta \text{ is the angle as shown}$$

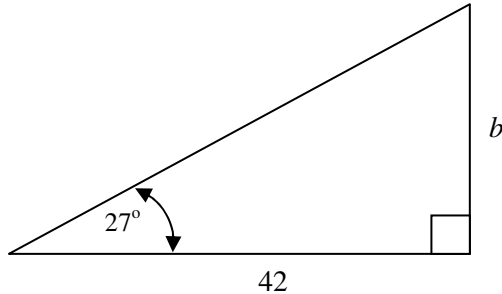
These ratios are abbreviated to  $\sin\theta$ ,  $\cos\theta$ , and  $\tan\theta$  respectively. A useful memory aid is Soh Cah Toa pronounced 'so-car-tow-a'

Unknown sides and angles in right angled triangles can be found using these ratios.

### Examples

Find the value of the indicated unknown (side length or angle) in each of the following diagrams.

(1)



#### Method

1. Determine which ratio to use.
2. Write the relevant equation.
3. Substitute values from given information.
4. Solve the equation for the unknown.

In this problem we have an angle, the **opposite** side and the **adjacent** side.  
The ratio that relates these two sides is the **tangent ratio**.

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

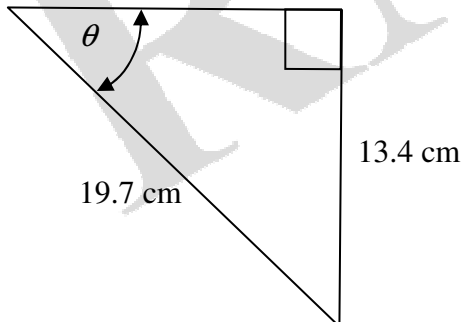
Substitute in the equation: (opposite side =  $b$ , adjacent side = 42, and  $\theta = 27^\circ$ )

$$\tan 27^\circ = \frac{b}{42} \quad \text{transpose to give}$$

$$b = 42 \times \tan 27^\circ$$

$$b = 21.4$$

(2)



In this triangle we know two sides and need to find the angle  $\theta$ .

The known sides are the **opposite** side and the **hypotenuse**.

The ratio that relates the **opposite** side and the **hypotenuse** is the **sine ratio**.

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{13.4}{19.7}$$

$$\sin \theta = 0.6082$$

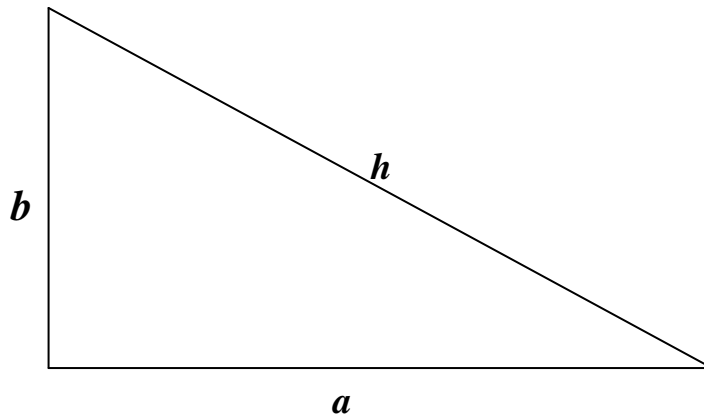
$$\text{opposite side} = 13.4\text{cm. hypotenuse} = 19.7\text{cm.}$$

This means we need the angle whose sine is 0.6082, or  $\sin^{-1} 0.6082$  from the calculator.

$$\therefore \theta = 42.9^\circ$$

## Pythagoras' Theorem

Pythagoras' Theorem states that in a right angled triangle the square of the length of the hypotenuse side ( $h$ ), is equal to the sum of the squares of the other two sides.

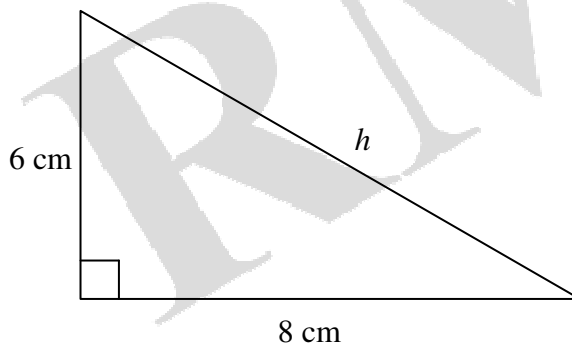


$$h^2 = a^2 + b^2$$

Pythagoras' Theorem can be used to find a side length of a right angled triangle given the other two side lengths

### Example 1

find the value of  $h$



$$h^2 = 6^2 + 8^2$$

$$\therefore h^2 = 36 + 64$$

$$\therefore h^2 = 100$$

$$\therefore h = 10$$

← Pythagoras' Theorem for this triangle

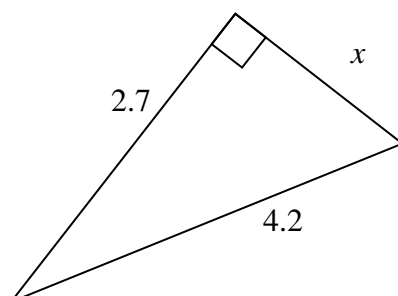
← square root of 100

### Note

Measurements must be in the same units and the unknown length will be in these same units - so  $h$  will be  $10\text{ cm}$

### Example 2

find the value of  $x$



$$4.2^2 = 2.7^2 + x^2$$

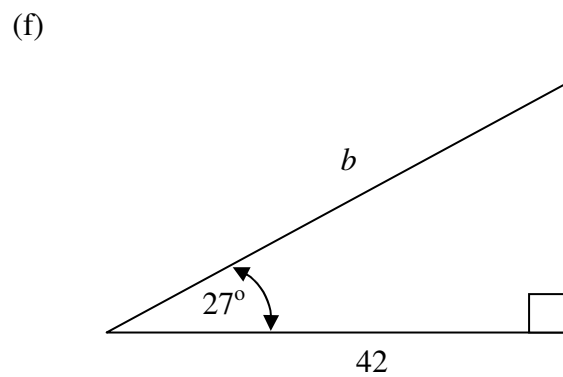
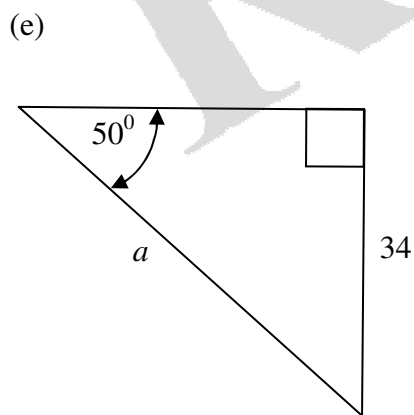
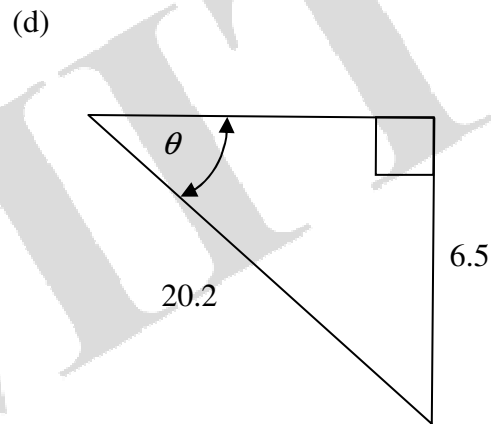
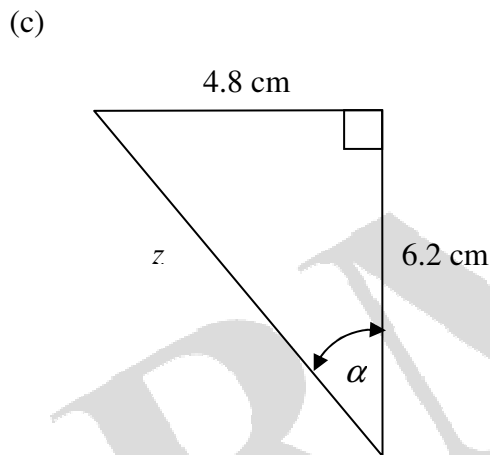
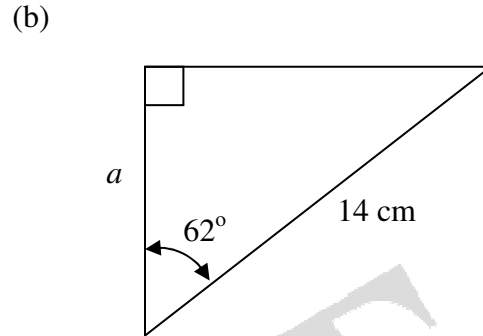
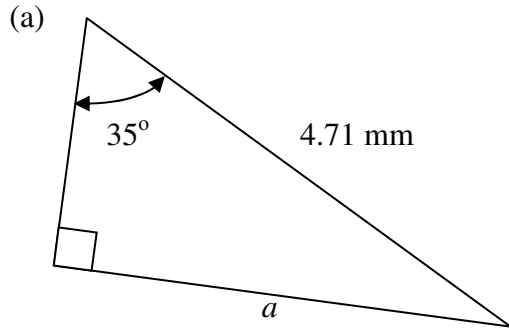
$$\therefore 17.64 = 7.29 + x^2$$

$$\therefore 10.35 = x^2$$

$$\therefore x = 3.22$$

**Exercise**

Find the value of the indicated unknown (side length or angle) in each of the following diagrams.



### Special angles and exact values

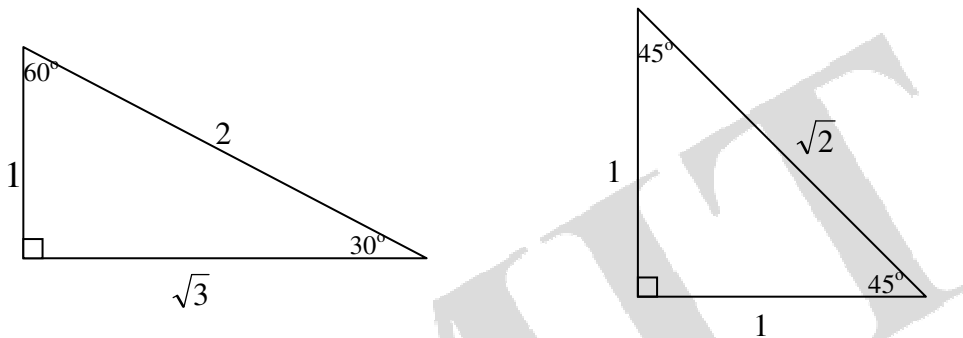
There are some special angles that enable us to obtain exact solutions for the functions sin, cos and tan.

If we take the two triangles below, and apply the basic trigonometry rules for sine, cosine and tangent –

$$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$$



From these two triangles, exact answers for sine, cosine and tangent of the angles  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  can be found.

$$\sin 45^\circ = \frac{1}{\sqrt{2}}, \quad \cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \tan 45^\circ = 1$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}, \quad \tan 60^\circ = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

### Answers

#### Exercise

- (a) 2.7mm      (b) 6.6cm      (c)  $z=7.8\text{cm}$ ,  $\alpha=37.7^\circ$       (d)  $18.8^\circ$       (e) 44.4      (f) 47.1