Scalars and Vectors

A vector is a quantity that has a magnitude and a direction. One example of a vector is velocity. The velocity of an object is determined by the magnitude (speed) and direction of travel. Other examples of vectors are force, displacement and acceleration.

A scalar is a quantity that has magnitude only. Mass, time and volume are all examples of scalar quantities.

Example 1.

If a car travels from point O to point A, which is 50km. away in a north-easterly direction, then the displacement of the car from O is 50km.NE. The displacement of the car is specified by the distance travelled (50km.) and the direction of travel (NE.) from O.

Displacement is therefore a vector, and the magnitude of the displacement (distance), is a scalar.

On the diagram below the displacement is represented by the directed line segment OA. The length of the line represents the magnitude of the displacement and is written |OA|.

The arrowhead represents the direction of the displacement.

Example 2.

A force of 50 Newton at an angle of 20° to the horizontal downward, is applied to a wheelbarrow. The diagram below shows a vector representing this force.

|AB| = 50N
**Geometric Vectors**

We will be considering vectors in three-dimensional space defined by three mutually perpendicular directions.

**Definitions and conventions.**

Vectors will be denoted by lower case bold letters such as \( \mathbf{a}, \mathbf{b}, \mathbf{c} \).

**Unit vectors \( \mathbf{i}, \mathbf{j}, \mathbf{k} \)**

Vectors with a magnitude of one in the direction of the \( x \)-axis, \( y \)-axis and \( z \)-axis will be denoted by \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k} \) respectively.

The notation \((a, b, c)\) will be used to denote the vector \((a \mathbf{i} + b \mathbf{j} + c \mathbf{k})\) as well as the coordinates of a point \(P(a, b, c)\). The context will determine which meaning is correct.

**Example**

In the diagram above the point \(P\) has coordinates \((3,4,5)\).

The vector \(\overrightarrow{OP}\) is the vector \(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}\). This may also be written \((3,4,5)\).
**Directed Line Segment.**

The directed line segment, or geometric vector, $\overrightarrow{PQ}$, from $P(x_1, y_1, z_1)$ to $Q(x_2, y_2, z_2)$ is found by subtracting the co-ordinates of $P$ (the initial point) from the co-ordinates of $Q$ (the final point).

$$\overrightarrow{PQ} = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$$

**Example.**

$$\overrightarrow{PQ} = (5 - 3)i + (6 - 4)j + (-1 - 1)k$$

$$\overrightarrow{PQ} = 2i + 2j - 2k$$

The directed line segment $\overrightarrow{PQ}$ is represented by the vector $2i + 2j - 2k$, or $(2, 2, -2)$. Any other directed line segment with the same length and same direction as $\overrightarrow{PQ}$ is also represented by $2i + 2j - 2k$ or $(2, 2, -2)$.

The directed line segment $\overrightarrow{QP}$ has the same length as $\overrightarrow{PQ}$ but is in the opposite direction.

$$\overrightarrow{QP} = -\overrightarrow{PQ} = -(2i + 2j - 2k) = -2i - 2j + 2k$$

or $(-2, -2, 2)$

**Position Vector.**

The position vector of any point is the directed line segment from the origin $O$ $(0,0,0)$ to the point and is given by the co-ordinates of of the point.

The position vector of $P(3, 4, 1)$ is $3i + 4j + k$, or $(3, 4, 1)$.

**Exercise 1.** Given the points $A(3,0,4)$ $B(-2, 4, 3)$ and $C(1, -5, 0)$, find:

(a) $\overrightarrow{AB}$  
(b) $\overrightarrow{AC}$  
(c) $\overrightarrow{CB}$  
(d) $\overrightarrow{BC}$  
(e) $\overrightarrow{CA}$

(f) The position vectors of $A$, $B$ and $C$.

Compare your answers 1(b) and 1(e), and 1(c) and 1(d). What do you notice?
**Length or Magnitude of a Vector.**

The length of a vector \( \mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k} \) is written \( |\mathbf{a}| \) and is evaluated by:

\[
|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}
\]

For example the length of the vector \( 2 \mathbf{i} + 2 \mathbf{j} - 2 \mathbf{k} \) equals \( \sqrt{2^2 + 2^2 + (-2)^2} = \sqrt{12} = 2\sqrt{3} \)

\( a_1, a_2, a_3 \) are often referred to as the components of vector \( \mathbf{a} \).

**Unit Vector**

A vector with a magnitude of one is called a unit vector. If \( \mathbf{a} \) is any vector then a unit vector parallel to \( \mathbf{a} \) is written \( \hat{\mathbf{a}} \) (a “hat”). The “hat” symbolises a unit vector.

Vector \( \mathbf{a} \) can then be written \( \mathbf{a} = |\mathbf{a}| \hat{\mathbf{a}} \)

therefore \( \hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|} \)

**Example**

A unit vector parallel to \( \mathbf{a} = (1, 2, 3) \) is the vector \( \hat{\mathbf{a}} = \frac{(1, 2, 3)}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{\sqrt{14}}(1, 2, 3) \)

**Adding and Subtracting Vectors.**

Vectors are added or subtracted by

- adding or subtracting their corresponding components
- using the triangle rule
- by using the parallelogram rule.

**Example**

If \( \mathbf{a} = (-3, 4, 2) \) and \( \mathbf{b} = (-1, -2, 3) \), find:

(i) \( \mathbf{a} + \mathbf{b} \) (ii) \( \mathbf{a} - \mathbf{b} \).

**Adding or subtracting components**

(i) \( \mathbf{a} + \mathbf{b} = (-3, 4, 2) + (-1, -2, 3) = (-4, 2, 5) \)

Similarly (ii) \( \mathbf{a} - \mathbf{b} = (-3, 4, 2) - (-1, -2, 3) = (-2, 6, -1) \)
Triangle Rule

(i) \( a + b \)

Place the tail of vector \( b \) at the head of vector \( a \) (point \( Q \)). The directed line segment \( \overrightarrow{PR} \) from the tail of vector \( a \) to the head of vector \( b \) is the vector \( a + b \).

(ii) To subtract \( b \) from \( a \), reverse the direction of \( b \) to give \( -b \) then add \( a \) and \( -b \).

\[
 a - b = a + (-b)
\]

Parallelogram Rule

(i) \( a + b \)

\( a \) and \( b \) are placed “tail-to-tail” (point \( P \)) and the parallelogram (PQRS) completed. The diagonal \( PR \) is the sum \( a + b \).

(ii) To find \( (a - b) \), reverse the direction of \( b \) to give \( -b \) then add \( a \) and \( -b \).

Exercise 2.

For vectors \( p \) (3, 6, 5), \( q \) (-4, 1, 0) and \( r \) (1, -3, 5) find:

(a) \( p + q \)  
(b) \( r + p \)  
(c) \( p - q \)
**Multiplication of a vector by a scalar.**

To multiply vector \( \mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k} \) by a scalar, \( m \), multiply each component of \( \mathbf{a} \) by \( m \).

\[
\mathbf{ma} = ma_1 \mathbf{i} + ma_2 \mathbf{j} + ma_3 \mathbf{k}
\]

The result is a vector of length \( |m| \times |\mathbf{a}| \)

If \( m > 0 \) the resultant vector is in the same direction as \( \mathbf{a} \)

If \( m < 0 \) the resultant vector is in the opposite direction from \( \mathbf{a} \).

Two vectors \( \mathbf{a} \) and \( \mathbf{b} \) are said to be parallel if and only if \( \mathbf{a} = k \mathbf{b} \) where \( k \) is a real constant.

**Example 1**

\( \mathbf{a} = (3 \mathbf{i} + \mathbf{j} - 2 \mathbf{k}) \) is multiplied by 7

\[
7\mathbf{a} = 7(3 \mathbf{i} + \mathbf{j} - 2 \mathbf{k}) = 21 \mathbf{i} + 7 \mathbf{j} - 14 \mathbf{k}.
\]

The magnitude of \( \mathbf{a} \) is

\[
|\mathbf{a}| = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14}
\]

\[
|7\mathbf{a}| = \sqrt{21^2 + 7^2 + (-14)^2} = \sqrt{686} = 7\sqrt{14} = 7|\mathbf{a}|
\]

**Example 2**

Find the value of \( m \) so that the vector \( \mathbf{a} \), \((4, m, 8)\) is parallel to the vector \( \mathbf{b} \), \((-6, 3, 12)\).

For \( \mathbf{a} \) and \( \mathbf{b} \) to be parallel \( \mathbf{a} = k \mathbf{b} \)

Therefore \((4, m, 8) = k (-6, 3, -12) = (-6k, 3k, -12k)\)

- equating “i”components
  \[4 = -6k\]
  \[k = \frac{-2}{3}\]

- equating “j”components
  \[m = 3k\]
  \[\therefore m = 3 \times \frac{-2}{3}\]
  \[m = -2\]

**Exercise 3**

Find the following

(a) \(3 \times (\mathbf{i} + 3 \mathbf{j} - 5 \mathbf{k})\)  
(b) \(8 \times (7 \mathbf{i} + 2 \mathbf{j} + 4 \mathbf{k})\)  
(c) \(-4 \times (\mathbf{j} - 3 \mathbf{k})\)
**Multiplication of a vector by a vector**

*(1) Dot product or scalar product*

The dot product of two vectors \( \mathbf{a} = (a_1, a_2, a_3) \) and \( \mathbf{b} = (b_1, b_2, b_3) \) is a scalar, defined by

\[
\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta , \quad \text{where } \theta \text{ is the angle between } \mathbf{a} \text{ and } \mathbf{b}
\]

and \( \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a} \)

*If \( \mathbf{a} \) is perpendicular to \( \mathbf{b} \) then \( \mathbf{a} \cdot \mathbf{b} = 0 \quad (\cos(\pi/2) = 0) \). In particular \( \mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0 \)

*If \( \mathbf{a} \) is parallel to \( \mathbf{b} \) then \( \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \quad (\cos(0)=1) \)

*In particular \( \mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1 \)

Also \( (a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}) \cdot (b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}) = \)

\[
(a_1 \cdot b_1) \mathbf{i} \cdot \mathbf{i} + (a_1 \cdot b_2) \mathbf{i} \cdot \mathbf{j} + (a_1 \cdot b_3) \mathbf{i} \cdot \mathbf{k} + (a_2 \cdot b_1) \mathbf{j} \cdot \mathbf{i} + (a_2 \cdot b_2) \mathbf{j} \cdot \mathbf{j} + (a_2 \cdot b_3) \mathbf{j} \cdot \mathbf{k} + (a_3 \cdot b_1) \mathbf{k} \cdot \mathbf{i} + (a_3 \cdot b_2) \mathbf{k} \cdot \mathbf{j} + (a_3 \cdot b_3) \mathbf{k} \cdot \mathbf{k}
\]

Thus \( \mathbf{a} \cdot \mathbf{b} \) can be defined by \( \mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \)

**Example 1**

Find the dot product of \( (2 \mathbf{i} + 3 \mathbf{j} + 4 \mathbf{k}) \) and \( (- \mathbf{i} - 2 \mathbf{j} + \mathbf{k}) \)

\[
(2 \mathbf{i} + 3 \mathbf{j} + 4 \mathbf{k}) \cdot (- \mathbf{i} - 2 \mathbf{j} + \mathbf{k}) = 2(-1) + 3(-2) + (4)(1) = -2 - 6 + 4 = -4
\]

\[
(2 \mathbf{i} + 3 \mathbf{j} + 4 \mathbf{k}) \cdot (- \mathbf{i} - 2 \mathbf{j} + \mathbf{k}) = -4
\]
**Example 2**

Find the scalar product of \( \mathbf{a} \) and \( \mathbf{b} \), as drawn, below where \( |\mathbf{a}| = \sqrt{14} \) and \( |\mathbf{b}| = \sqrt{6} \)

\[
\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta
\]
\[
= \sqrt{14} \times \sqrt{6} \times \cos 30^\circ
\]
\[
= \sqrt{14} \times \sqrt{6} \times \frac{\sqrt{3}}{2}
\]
\[
= 3\sqrt{7}
\]
\[
\mathbf{a} \cdot \mathbf{b} = 3\sqrt{7}
\]

**Exercise 4** Find the dot product of the following vectors:

(a) \( 3\mathbf{i} \) and \( 5\mathbf{j} \)  
(b) \( 2\mathbf{i} + 3\mathbf{k} \) and \( 7\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} \) 
(c) \( 5\mathbf{k} \) and \( \mathbf{j} - 2\mathbf{k} \)  
(d) \( (2, 0, 4) \) and \( (-3, 1, 3) \) 
(e) \( (0, 5, 1) \) and \( (4, 0, 0) \) 
(f) \[
\begin{align*}
\mathbf{a} \cdot \mathbf{b} &= |\mathbf{a}| |\mathbf{b}| \cos \theta \\
&= \sqrt{14} \times \sqrt{6} \times \cos 30^\circ \\
&= \sqrt{14} \times \sqrt{6} \times \frac{\sqrt{3}}{2} \\
&= 3\sqrt{7} \\
\end{align*}
\]

(2) **Cross product or vector product**

The cross product of two vectors \( \mathbf{a} \) and \( \mathbf{b} \) is the vector \( \mathbf{a} \times \mathbf{b} \), which is perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \) and is given by

\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3
\end{vmatrix}
\]

The magnitude of \( \mathbf{a} \times \mathbf{b} \) is given by \( |\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta \) where \( \theta \) is the angle between \( \mathbf{a} \) and \( \mathbf{b} \).

The direction of \( \mathbf{a} \times \mathbf{b} \) is that in which your thumb would point if the fingers of your right are curled from \( \mathbf{a} \) to \( \mathbf{b} \).

*In particular* \( \mathbf{i} \times \mathbf{j} = \mathbf{k}, \quad \mathbf{j} \times \mathbf{k} = \mathbf{i}, \quad \mathbf{k} \times \mathbf{i} = \mathbf{j} \)

\( \mathbf{i} \times \mathbf{k} = -\mathbf{j}, \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i}, \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \)
If \(a\) is parallel to \(b\) then \(a \times b = 0\). (\(\sin 0^\circ = 0\))

In particular \(i \times i = j \times j = k \times k = 0\)

If \(a\) is perpendicular to \(b\) then \(|a \times b| = |a||b|\) (\(\sin 90^\circ = 1\))

\(a \times b = -b \times a\) (the cross product is not commutative.)

**Example 1**

Find \(a \times b\) if \(a = 2i + j + k\) and \(b = 5j + 3k\)

\[
\begin{vmatrix}
  i & j & k \\
  2 & 3 & 1 \\
  0 & 5 & 3
\end{vmatrix} = (9 - 5)i - (6 - 0)j + (10 - 0)k
\]

\(a \times b = 4i - 6j + 10k\)

**Example 2**

Find \(a \times b\) if \(a = (2,1,1)\) and \(b = (-2,4,0)\)

\[
\begin{vmatrix}
  i & j & k \\
  2 & 1 & 1 \\
  -2 & 4 & 0
\end{vmatrix} = (0 - 4)i - (0 + 2)j + (8 + 2)k
\]

\(a \times b = -4i - 2j + 10k\)

**Example 3**

Find \(a \times b\) if \(a = (2,1,1)\) and \(b = (8,4,4)\)

Because \(a = 4b\), \(a\) is parallel to \(b\) therefore \(a \times b = 0\)

**Exercise 5** Find the cross product of the following vectors:

(a) \(j \times k\)  
(b) \(i \times 4i\)  
(c) \((2i + 3j - k) \times (3j + 2k)\)

(d) \(3j \times 5i\)  
(e) \((i - 3j + k) \times (2i + j - k)\)
**Projection of vectors.**

Consider the diagram below:

![Diagram of vectors PQ and PS](image)

Let \( \overrightarrow{PQ} = \mathbf{a} \) and \( \overrightarrow{PS} = \mathbf{b} \).

**Scalar projection**

The scalar projection of vector \( \mathbf{a} \) in the direction of vector \( \mathbf{b} \) is the length of the straight line \( \overline{PR} \) or \( \overline{PR} \).

\[
\overline{PR} = |\mathbf{a}| \cos \theta. \quad \text{Also} \quad \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \quad \text{(because} \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta) \]

Therefore

\[
\overline{PR} = \left(|\mathbf{a}|\right) \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|} = \mathbf{a} \hat{\mathbf{b}} \quad \text{(cancel} |\mathbf{a}|, \text{and use} \quad \frac{\mathbf{b}}{|\mathbf{b}|} = \hat{\mathbf{b}} \).
\]

The scalar projection of a vector \( \mathbf{a} \) in the direction of vector \( \mathbf{b} \) is given by

\[
\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|} = \mathbf{a} \hat{\mathbf{b}} \quad \text{or} \quad |\mathbf{a}| \cos \theta
\]

**Vector projection**

The vector projection of vector \( \mathbf{a} \) in the direction of vector \( \mathbf{b} \) is a vector in the direction of \( \mathbf{b} \) with a magnitude equal to the length of the straight line \( \overline{PR} \) or \( |\overline{PR}| \).

Therefore the vector projection of \( \mathbf{a} \) in the direction of \( \mathbf{b} \) is the scalar projection multiplied by a unit vector in the direction of \( \mathbf{b} \).

The vector projection of vector \( \mathbf{a} \) in the direction of vector \( \mathbf{b} \) is given by

\[
(\mathbf{a} \cdot \hat{\mathbf{b}}) \hat{\mathbf{b}} = \frac{(\mathbf{a} \cdot \mathbf{b}) \mathbf{b}}{|\mathbf{b}|^2}
\]
**Angle between two vectors**

The angle, \( \theta \) between two vectors can be found from the definition of the dot product

\[
\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta
\]

therefore \( \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} \)

\( \theta \) can also be found from \( \cos \theta = \frac{\mathbf{a} \cdot \hat{\mathbf{b}}}{|\mathbf{a}|} \)

**Example**

Find:
(a) the scalar projection of vector \( \mathbf{a} = (2, 3, 1) \) in the direction of vector \( \mathbf{b} = (5, -2, 2) \).
(b) the angle between \( \mathbf{a} \) and \( \mathbf{b} \).
(c) the vector projection of \( \mathbf{a} \) in the direction of \( \mathbf{b} \).

(a) **Scalar projection**

\[
|\mathbf{b}| = \sqrt{25 + 4 + 4} = \sqrt{33}
\]

therefore \( \hat{\mathbf{b}} = \frac{(5, -2, 2)}{\sqrt{33}} \)

\[
\mathbf{a} \cdot \hat{\mathbf{b}} = (2, 3, 1) \cdot \frac{(5, -2, 2)}{\sqrt{33}} = \frac{10 + (-6) + 2}{\sqrt{33}} = \frac{6}{\sqrt{33}}
\]

The scalar projection of \( \mathbf{a} \) in the direction of \( \mathbf{b} \) is \( \frac{6}{\sqrt{33}} \)

(b) **Angle between \( \mathbf{a} \) and \( \mathbf{b} \)**

The scalar projection of \( \mathbf{a} \) in the direction of \( \mathbf{b} \) is also equal to \( |\mathbf{a}| \cos \theta \), where \( \theta \) is the angle between \( \mathbf{a} \) and \( \mathbf{b} \).

Therefore \( \frac{6}{\sqrt{33}} = |\mathbf{a}| \cos \theta \)

\[
|\mathbf{a}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}
\]

\[
\therefore \frac{6}{\sqrt{33}} = \sqrt{14} \cos \theta
\]

\[
\therefore \cos \theta = \frac{6}{\sqrt{33 \times 14}} = 0.2791
\]

\( \therefore \theta = 74^\circ \)

The angle between \( \mathbf{a} \) and \( \mathbf{b} \) is \( 74^\circ \).
(c) **Vector projection**

The vector projection $\mathbf{a}$ in the direction of $\mathbf{b}$ equals:

\[
\text{(scalar projection of } \mathbf{a} \text{ in the direction of } \mathbf{b}) \hat{\mathbf{b}} = \frac{6}{\sqrt{33}} \times \frac{(5, -2, 2)}{\sqrt{33}} = \frac{6(5, -2, 2)}{33}
\]

The vector projection of $\mathbf{a}$ in the direction of $\mathbf{b}$ is $\frac{6(5, -2, 2)}{33}$

**Exercise 6** For the following pairs of vectors find:

(i) the scalar projection of $\mathbf{a}$ on $\mathbf{b}$

(ii) the angle between $\mathbf{a}$ and $\mathbf{b}$

(iii) the vector projection of $\mathbf{a}$ on $\mathbf{b}$

(a) $\mathbf{a} = (2, 3, 1)$ $\mathbf{b} = (5, 0, 3)$

(b) $\mathbf{a} = (0, 0, 3)$ $\mathbf{b} = (0, 0, 7)$

(c) $\mathbf{a} = (5, 0, 0)$ $\mathbf{b} = (0, 3, 0)$

(d) $\mathbf{a} = (-3, 2, -1)$ $\mathbf{b} = (2, 1, 2)$
Answers

1. (a) $(-5, 4, -1)$  
   (b) $(-2, -5, -4)$  
   (c) $(-3, 9, 3)$  
   (d) $(3, -9, -3)$  
   (e) $(2, 5, 4)$  
   (f) $\overrightarrow{OA} = 3i + 4k, \quad \overrightarrow{OB} = -2i + 4j + 3k, \quad \overrightarrow{OC} = i - 5j$

2. (a) $(-1, 7, 5)$  
   (b) $(4, 3, 10)$  
   (c) $(7, 5, 5)$

3. (a) $3i + 9j - 15k$  
   (b) $56i + 16j + 32k$  
   (c) $-4j + 12k$

4. (a) $0$  
   (b) $26$  
   (c) $-10$  
   (d) $6$  
   (e) $0$  
   (f) $10\sqrt{2}$  
   (g) $0$

5. (a) $i$  
   (b) $0$  
   (c) $9i - 4j + 6k$  
   (d) $-15k$  
   (e) $-i + 9j + 7k$

6. (a)(i) $\frac{13}{\sqrt{34}}$  
   (ii) $53^0$  
   (iii) $\frac{13}{34}(5, 0, 3)$  
   (b)(i) $3$  
   (ii) $0^0$  
   (iii) $\frac{3}{7}(0, 0, 7)$  
   (c)(i) $0$  
   (ii) $90^0$  
   (iii) $0$  
   (d)(i) $-2$  
   (ii) $122^0$  
   (iii) $\frac{-2}{3}(2, 1, 2)$